

UNIVERSITI TEKNOLOGI MARA

**STABILITY ANALYSIS OF TWO-
SPECIES MUTUALISM MODEL
WITH TIME DELAY AND
HARVESTING**

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of the requirements for the degree of
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AUTHOR'S DECLARATION

I declare that the work in this thesis was carried out in accordance with the regulations of Universiti Teknologi MARA. It is original and is the result of my own work, unless otherwise indicated or acknowledged as referenced work. This thesis has not been submitted to any other academic institution or non-academic institution for any degree or qualification.

I, hereby, acknowledge that I have been supplied with the Academic Rules and Regulations for Post Graduate, Universiti Teknologi MARA, regulating the conduct of my study and research.


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ABSTRACT

In this thesis, we analyze a two-species mutualism model with and without time delay and harvesting functions. The time delay is considered in the model to improve accuracy because the growth rate of the population depends not only on the present size of population but also on past information while the harvesting function is introduced to describe the rate of removal of the species. The stability of the model involving time delay is determined by defining a time delay margin. The global stability analysis of the unique positive equilibrium point of mutualism model with two different harvesting functions is proven by constructing a suitable Lyapunov function. The two harvesting functions are harvesting at a constant rate and harvesting at a rate proportional to the population size. The effect of time delay and two different harvesting functions on the dynamics of the population has been examined. We consider time delay into the basic model without harvesting and discuss the effect of delay on the stability of the unique positive equilibrium point. From the analysis, it can be concluded that time delay can induce instability of a stable equilibrium point. The time delay model is then modified to include a harvesting function. Then we discuss the effect of harvesting and time delay on the dynamics of the model. The analysis shows that when there is no delay, varying the harvesting rate does not affect the stability of the model if the value of harvesting rate is under control while time delay can cause a stable equilibrium point to become unstable.

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